Math 131A-1: Homework 2

Due: January 16, 2015

1. Read Sections 4-5 in Ross.

2. Do problem 2.3, 3.4, 3.7, and 3.8 in Ross.

3. Do problems 4.1 - 4.4 in Ross for (a), (b), (m), (r), and (w). [Please do not use the answer formats suggested by the textbook; instead use complete sentences and standard capitalization.]

4. Let F be a field; that is, F is a set with two operations $+$ and $\times$ obeying the nine field axioms introduced in class.

   (a) Show that the additive identity 0 postulated by axiom (A3) is unique; that is, show that if there is another element $0'$ satisfying $a + 0' = a$ for all a in F, then $0' = 0$. Show also that for each $a \in F$, the additive inverse $-a$ is unique.

   (b) Show that the multiplicative identity 1 postulated by axiom (M3) is unique, and that for each nonzero $a \in F$, the multiplicative inverse a^{-1} is unique.

5. Recall that the complex numbers $\mathbb{C}$ are the set of all numbers $a + bi$ such that $a, b \in \mathbb{R}$ and i is a number satisfying $i^2 = -1$. The operations of addition and multiplication on $\mathbb{C}$ are as follows:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$$

   (a) Show that $\mathbb{C}$ is a field.

   (b) Show there is no relation $\leq$ on $\mathbb{C}$ which makes $\mathbb{C}$ into an ordered field.